LJAST
Libyan Journal of Applied Science and Technology
مجلة ليبيا للعلوم التطبيقية والتقنية

Volume 13 Issue 01
June 2025
ISSN 2958-6119

Development of a Combined (Parallel & Parabolic) Rheomatic Device for Determination of Pressure Dependent Viscosity

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Abstract

Hydrodynamic pressure technique is a relatively new and innovative technique for rheological studies of viscous Newtonian fluids. These principles extensively used for the last twenty years for drawing and coating of strips and wires. The Rheometric Device consists of a rotating inner cylinder (rod) in a fixed hollow outer cylinder. The complex geometry gap between the two cylinders filled with a viscous non-Newtonian fluid. When the rod rotates inside, while the hollow cylinder filled with a viscous fluid, shearing takes place and hydrodynamic pressure develops, the magnitude of which is dependent on the shape of the surfaces, the viscosity of the fluid as well as the shear rate , the speed with which the inner solid rod is rotating. The Rheometer has been developed to determine the Rheological properties of fluid at pressures of *up to 100 bar* and a shear rate range of $500\ to$ $400\ sec{-}1$.
Theoretical model developed based on Newtonian characteristics, shear rate viscosity relationship was determined using the Rheometer (combined parallel and parabolic unit) at different pressures by comparing the calculated theoretical pressure distribution with the experimental results.

Keywords: Newtonian fluid, Polymer, Rotating speed, Parabolic, Shear rate viscosity.

INTRODUCTION:

In Plasto-hydrodynamic die-less wire drawing process, a pressure unit of certain internal geometry replaces the conventional reduction die. The deformation induced by the combined effect of the hydrodynamic pressure and stress generated in the unit due to the motion of the wire, the viscous action of the polymer melt and the pulling action of the wire. The dimensions of the unit are such that the smallest bore size is greater than the incoming wire diameter. In this system, larger magnitude of the hydrodynamic pressure is advantageous in obtaining greater deformation. Different studies, carried out by researchers, using different types of pressure units, such as the complex pressure unit, the parallel bore pressure unit, the simple tapered pressure unit were the effect of different parameters affected the process studded for example the drawing force, the units internal dimensions, the assumption of non-Newtonian pressure fluid was also verified, those studies published on different journal and conferences.

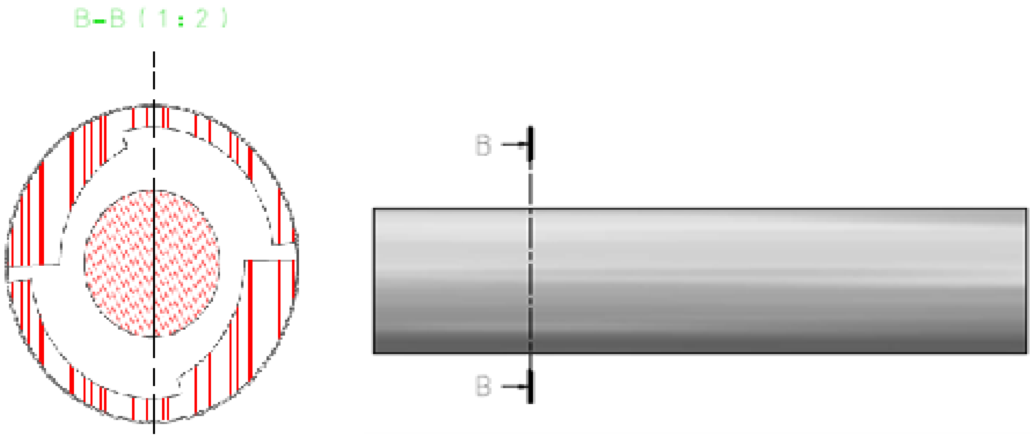

Figure (1) combined pressure unit (parallel and parabolic bore)

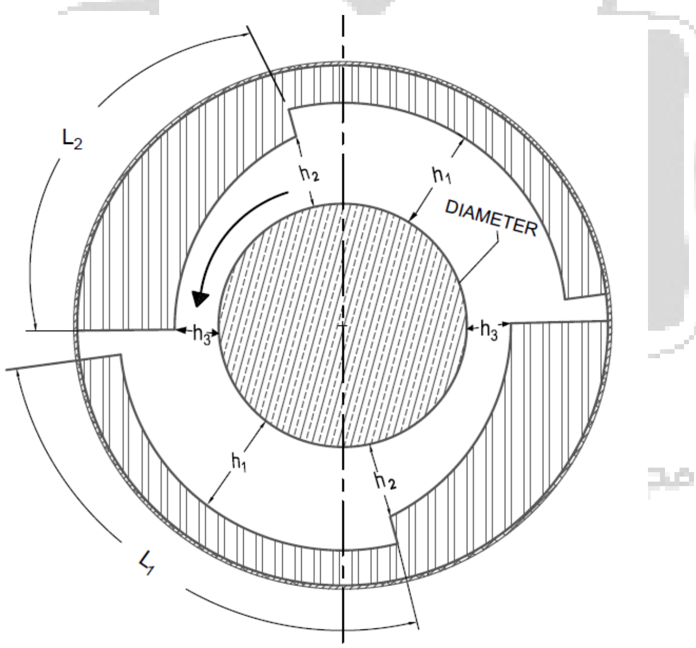

Figure (2) Pressure Unit Diminutions

Pressure Modeling within the unit:

1- Parallel part of the unit:

The relationship between the pressure and shear stress gradient between the outer surface of the rotating bar and the inner surface of the unit given by:

$$\left(\frac{dP}{dx}\right)_1 = \left(\frac{d\tau_{xy}}{dy}\right)_1 = \left(\frac{d\tau}{dy}\right)_1 \qquad (1)$$

The relationship between the shear stress and the rate of the shear for the Newtonian fluid given by:

$$\tau_1 = \mu \left(\frac{du}{dy}\right)_1 \qquad (2)$$

Where μ the viscosity and u_1 is the fluid velocity at a distance y from the surface of the rotating bar. Integrating equation (1) with respect to y and noting that (dP/dx) is constant with y we have:

$$\tau_1 = p_1' \, y + \tau c_1 \qquad (3)$$

Where $p' = (dP/dx)$ and $\boldsymbol{\tau c_1}$ is the shear stress on the rotating bar surface at $y = 0$, substituting for $\boldsymbol{\tau_1}$ from equation **(2)** into equation **(3)** we have:

$$\mu \left(\frac{du}{dy}\right)_1 = p_1' \, y + \tau c_1$$

Which after integration becomes,

$$\mu u_1 = (p_1' y^2)/2 + \tau c_1 y + c_1$$

$$u_1 = (p_1' y^2)/2\,\mu + \tau c_1 y/\mu + c_1/\mu \qquad (4)$$

Applying the boundary condition that at $y = 0$ (at the surface of the rotating wire) $u_1 = \omega_1$

We have $C_1 = \mu\, \omega_1$

So that $u_1 = (p_1' y^2)/2\,\mu + (\tau c_1 y)/\mu + \omega_1$ (5)

Applying the boundary conditions that at $y = h_1$(at the surface of the unit) u_1= zero in equation (5) and rearranging we have:

$$\tau c_1 = -(p_1' \, y^2)/2\mu \; - (\mu\omega_1)/h_1$$

$$\tau c_1 = -(p_1' \, h_1)/2 \; - \; (\mu\omega_1)/h_1 \qquad (6)$$

The flow of the pressure medium in the *first part* of the unit Q_1 , is given by:

$$Q_1 = \int_0^{h_1} u_1 \; dy$$

Which upon substituting for $\boldsymbol{u_1}$ from equation **(5)** we get:

$$Q_1 = \int_0^{h_1} ((p_1' y^2)/2\,\mu + (\tau c_1 y)/\mu + \omega_1)\ dy$$

The above, after integration becomes,

$$Q_1 = (p_1'\, h_1^3)/6\mu + \omega_1 h_1 + (\tau c_1 h_1^2)/2\mu$$

Substituting for $\boldsymbol{\tau c_1}$ from equation (6) into the above equation, we get:

$$Q_1 = -(p_1' h_1^3)/12\mu + (\omega_1 h_1)/2 \qquad (7)$$

Where $p_1' = (dP/dx)$.

The continuity of fluid flow gives: $(dQ/dx) + (dQ/dy) + (dQ/dz) =$ zero

But $(dQ/dy) = (dQ/dz) = 0$,

And Hence, $(dQ/dx) = 0$

Therefore, for a given h_1, h_2 and v_1 , (dQ/dx) in equation (7) must be constant.

Therefore:

$$(dP/dx) = p_1' = P_{ms}/L_1 = \text{constant.}$$

P_{ms} Is the pressure at the step and L_1 is the length of the first part of the unit. It shows that the pressure profile in the parallel part of the unit is linear.

The pressure at any point $x_1 < L_1$ given by:

$$P = \int_0^{x_1} p_1'\, dx = (P_{ms}/L)\, x_1 \qquad (8)$$

When $x_1 = L_1$ So

$$P_{ms} = \frac{\left\{\left[\left(\frac{6\mu\omega h_1}{2B}\right)\left(\frac{1}{(h_3)^2} - \frac{1}{(h_2)^2}\right)\right] + \left(\frac{6\mu\omega}{B}\right)\left(\frac{1}{h_2} - \frac{1}{h_3}\right)\right\}}{\left\{1 + \left(\frac{(h_1)^3}{2BL_1}\right)\left(\frac{1}{(h_3)^2} - \frac{1}{(h_2)^2}\right)\right\}} \qquad (9)$$

From the geometrical configuration of the second part of the unit, the gap between the second part of the unit and the surface of the rotating bar given by:

$$h = h_2 - Bx_2\ ,\ \ Where \qquad B = \left(\frac{h_2 - h_3}{L_2}\right)$$

Then:

$$P_{max} = \left(\frac{6\mu\omega}{B}\right)\left(\frac{1}{2\bar{h}} - \frac{1}{h_3} - \frac{\bar{h}}{2(h_3)^2}\right) \quad (10)$$

Where:

$$\bar{h} = h_1 - \left(\frac{(h_1)^3 Pms}{6\mu v L_1}\right) \quad (11)$$

$\bar{h}$ The height at which maximum pressure occur.

The axial stress on the rotating bar at any point distance x_1 from the entry obtained by considering the shear force action on the surface of the rotating bar Thus,

$\sigma_{x_1} = (\tau c_1 x_1)/D$

Substituting $\boldsymbol{\tau c_1}$ from equation **(6),** we get:

$$\sigma_{x_1} = \left(-\frac{x_1}{D}\right)\left[\left(\frac{P_{ms}\,h_1}{2L_1}\right) + \left(\frac{\mu\omega_1}{h_1}\right)\right] \quad (12)$$

Parabolic part of the unit:

For Newtonian fluid, the pressure gradient given as:

$$\left(\frac{dP}{dx}\right)_2 = \left(\frac{d\tau}{dy}\right)_2 \quad (13)$$

In addition, the shear stress expressed by:

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 \quad (14)$$

Where:

$\boldsymbol{u_2}$ Is the velocity of the fluid in the parabolic part of the unit,

Differentiating the above equation (14) with respect to y, we obtain,

$$\left(\frac{d\tau}{dy}\right) = \mu\left(\frac{d^2u_2}{dy^2}\right) \quad (15)$$

Substituting the above equation in **(13),** we obtain,

$$\left(\frac{d^2u_2}{dy^2}\right) = \frac{1}{\mu}\left(\frac{dP}{dx}\right) \qquad (16)$$

Integrating and noting that (dP/dx) assumed to be constant with $\boldsymbol{y}$ to get:

$$\left(\frac{du_2}{dy}\right) = \frac{1}{\mu}\left(\frac{dP}{dx}\right) y + C_2$$

Integrating again,

$$u_2 = \frac{1}{2\mu}\left(\frac{dP}{dx}\right) y^2 + y\, C_2 + C_3$$

Where C_2 and C_3 are constants.

Using the boundary conditions that $u_2 = 0$ at $y = h$ (at the surface of the unit), and $u_2 = \omega_2$ at $y = 0$ (at the surface of the rotating bar) we obtain,

$$C_2 = \frac{1}{2\mu}\left(\frac{dP}{dx}\right) h - \frac{\omega_2}{h}$$

Moreover $\boldsymbol{C_3 = \omega_2}$ so that:

$$u_2 = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)(y^2 - hy) + \omega_2\left(1 - \left(\frac{y}{h}\right)\right) \qquad (17)$$

The flow of the fluid in axial direction given by,

$$Q_x = \int_0^h u_2 dy$$

Which upon substituting for $\boldsymbol{u_2}$ from equation **(17)** and integrating becomes,

$$Q_x = \left(-\frac{h^3}{12\mu}\right)\left(\frac{dP}{dx}\right) + \left(v_2 \frac{h}{2}\right) \qquad (18)$$

The continuity of the fluid flow shows that:

$$\left(\frac{dQ_x}{dx}\right) + \left(\frac{dQ_y}{dy}\right) + \left(\frac{dQ_z}{dz}\right) = 0$$

According to the assumptions made,

$(dQ_y/dy) = (dQ_z/dz) = 0$, and Hence $(dQ_x/dx) = 0$

Differentiating equation (18) with respect to x and equating to ZERO, than integrating we obtain,

$$\left(\frac{h^3}{6\mu}\right)\left(\frac{dP}{dx}\right) = \omega_2\, h + C_4 \qquad (19)$$

Using the boundary conditions that maximum pressure $(dP/dx) = 0$ occurs at point where $h = h_b$ we have $C_4 = -\omega_2\, h$,

So that:

$$\left(\frac{dP}{dx}\right) = 6\mu\omega_2\left(\frac{1}{h^2} - \frac{h}{h^3}\right)$$

Let,

$$H(x) = \int_0^h \frac{1}{h^2}\, dx \quad , \quad G(x) = \int_0^h \frac{1}{h^3}\, dx$$

For the converging parabolic unit, the unit geometry considered as,

$$h(x) = -a^2\,(x+b)^2 + C^2$$

Substituting $h(x)$ in the expression for $H(x)$ and $G\,(x)$ it becomes,

$$H(x) = \int_0^x \frac{1}{[-a^2(x+b)^2 + C^2]^2}\, dx \qquad (20\_a)$$

$$G(x) = \int_0^x \frac{1}{[-a^2(x+b)^2 + C^2]^3}\, dx \qquad (20\_b)$$

After integration of equation (20, a-b), we get:

$$H(x) = \frac{1}{4aC^3}\left[\ln\left[\frac{x+b+d}{x+b-d}\right] - \ln\left[\frac{b+d}{b-d}\right] - d\left[\frac{1}{x+b+d} + \frac{1}{x+b-d} - \frac{1}{b+d} - \frac{1}{b-d}\right]\right] \qquad (21)$$

And

$$G(x) = \frac{1}{16d^5\, a^6}\left[3\ln\left[\frac{x+b+d}{x+b-d}\right] - 3\ln\left[\frac{b+d}{b-d}\right] - 3d\left[\frac{1}{x+b+d} + \frac{1}{x+b-d} - \frac{1}{b+d} - \frac{1}{b-d}\right] - d^2\left[\frac{1}{(x+b+d)^2} - \frac{1}{(x+b-d)^2} - \frac{1}{(b+d)^2} + \frac{1}{(b-d)^2}\right]\right] \qquad (22)$$

The boundary conditions for the second part are:

(i) $x = 0,$ $\quad$ P = 0

(ii) $x = L$ $\quad$ P = 0

Therefore:

(i) with the boundary conditions $(\boldsymbol{i})$, the pressure expression becomes:

$$P(x) = 6\mu\omega[H(x) - H_b G(x)]$$

(ii) with the boundary conditions $(\boldsymbol{ii})$,the pressure profile in the unit is:

$$H_b = \frac{H(L)}{G(L)}$$

Therefore, the pressure profile in this unit is:

$$\mathrm{P(x)} = 6\mu\omega\left[\mathrm{H(x)} - \frac{\mathrm{H(L)G(x)}}{\mathrm{G(L)}}\right] \qquad (23)$$

To obtain the position where optimum pressure occur it is necessary to calculate $\boldsymbol{G(L)}$ and $\boldsymbol{H(L)}$ for hydrodynamic converging parabolic part.

$$h(x) = -\left(h_2 - \frac{h_3}{L_2}\right)x^2 + h_2$$

Where: $\boldsymbol{d = c/a}$

Therefore in this case the terms a could be substituted for:

1- $a = \left(\frac{(h_2 - h_3)^{\frac{1}{2}}}{L_2}\right).$ $\qquad$ 2- $b = 0.$ $\qquad$ 3- $C = (h_2)^{\frac{1}{2}}.$

And substituting the values for $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ at $\boldsymbol{x = L}$, the die geometry can be expressed as,

$$H(L) = \frac{L_2}{4h_2\sqrt{h_2}\sqrt{h_2 - h_3}}\left[\ln\frac{\sqrt{h_2 - h_3} + \sqrt{h_2}}{\sqrt{h_2 - h_3} - \sqrt{h_2}}\right] + \frac{L_2}{2h_2h_3} \qquad (24)$$

In the same way

$$G(L) = \frac{1}{16h_2^2\sqrt{h_2}\sqrt{h_2-h_3}}\left[3\ln\frac{\sqrt{h_2-h_3}+\sqrt{h_2}}{\sqrt{h_2-h_3}-\sqrt{h_2}} - 3d\left[\frac{2L_2}{L_2^2-d^2)} + d^2\left(\frac{4dL_2}{L_2^2-d^2)^2}\right)\right]\right]$$

Therefore,

$$G(L) = \frac{L_2}{16h_2^2\sqrt{h_2}\sqrt{h_2-h_3}}\left[3\ln\frac{\sqrt{h_2-h_3}+\sqrt{h_2}}{\sqrt{h_2-h_3}-\sqrt{h_2}}\right] + \frac{1}{16h_2}\left[\frac{6L_2}{h_2h_3} + \frac{4L_2}{h_3^2}\right] \quad (25)$$

Nomenclature

D	Rotating bar diameter.
Q_1	Flow of fluid in the first part (parallel part|) of the unit.
Q_2	Flow of fluid in the second part of the unit.
$P' = dP/dx$	Pressure gradient in the unit.
τ	Shear stress of the polymer melt in the unit.
τ_C	Shear stress on the rotating bar in the pressure unit.
μ	Viscosity of the fluid.
ω	Velocity of the rotating bar
σ_x	Axial stress.
P_{ms}	Pressure at the step.
P_{max}	Maximum pressure in the pressure unit.
L_1	Length of the first part of the unit.
L_2	Length of the parabolic part "second part" of the unit.
u	Velocity of the fluid.
$P(x)$	Pressure expression.
H_b	The position of optimum pressure.

h_1	Gap at the entry end of the first unit.
h_2	Gap at the step.
h_3	Gap at the end of the second unit.

Results & Discussion:

Results obtained using Excel programme after feeding the equations and iterating them. Fig.(1) shows the effect of viscosity change on maximum pressure generated within the unit for constant rotation speed of 20 rad/sec, For refernce viscosity of $\mu = 50\text{N.S}/m^2$ maximum pressure was $11\text{MN}/m^2$, we ara changed refernce viscosity of $\mu = 100\text{N.S}/m^2$ it increses to $21\text{MN}/m^2$, again we ara changed refernce viscosity of $\mu = 150\text{N.S}/m^2$ it increses to $33\text{MN}/m^2$.

We note in the third cases, the maximum value of the pressure was recorded at gap ${}^{h_2}/_{h_3} = 2$, and the maximum value of the pressure $\mu = 150\text{N.S}/m^2$ and equal $33\text{MN}/m^2$.
Also, when viscosity is increased, the value of the maximum pressure increases and is a positive relationship.

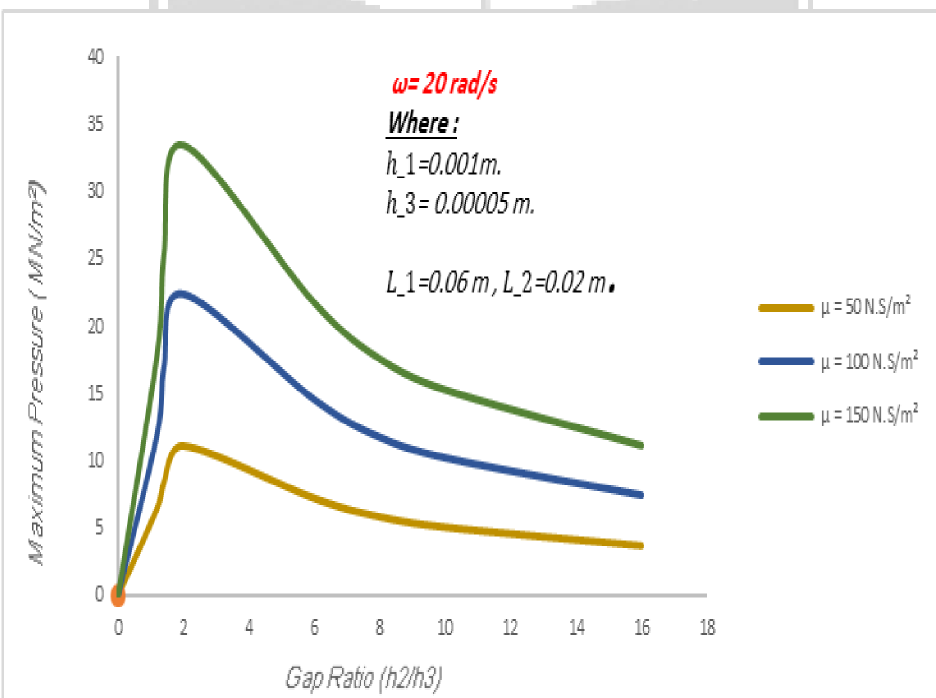

Fig (3): shows the gap ratio and pressure distribution.

Fig.(3) shows the results of gap ratio effect on pressure distribution within the unit at different viscosities where At a fixed rotating speed $\omega = 20\ rad/s$, the maximum pressure value was found at the same length L = 0.078 m

In Viscosity 50 the maximum value of the pressure is 5MN/m^2, when we changed the viscosity to 100 the maximum value of the pressure was11MN/m^2, when changed to 150 we find that the maximum value of the compression is 22MN/m^2.

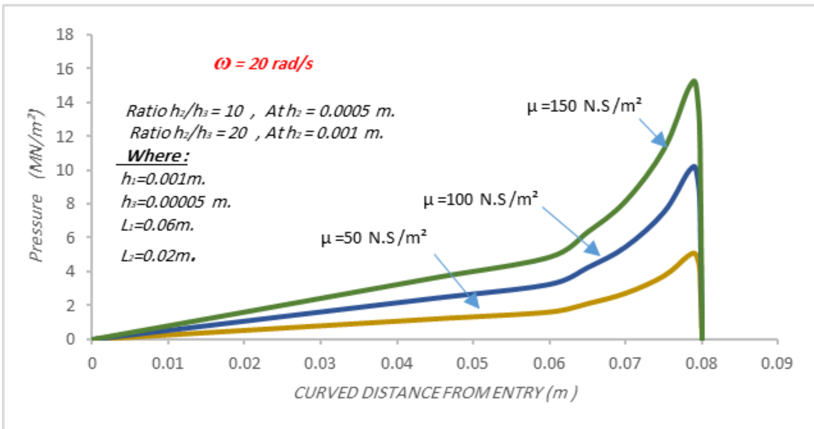

Fig. (4) show the pressure distribution for two different length ratios.

Fig.(4) indicates the effect of changing the the pressure distribution for two different length ratios ,we can noted the effect of length ratio on pressure distribution ratios at L=0.079 m, $^{L_1}/_{L_2}$ = 3 & $^{L_1}/_{L_2}$ = 3.25 at the rotating speed of of $\omega = 20\ rad/s,$ and $L_2\ constant = 0.02\ m\ (20mm)$
This figure suggests that the change of length ratios, where the maximum pressure value is almost the same.

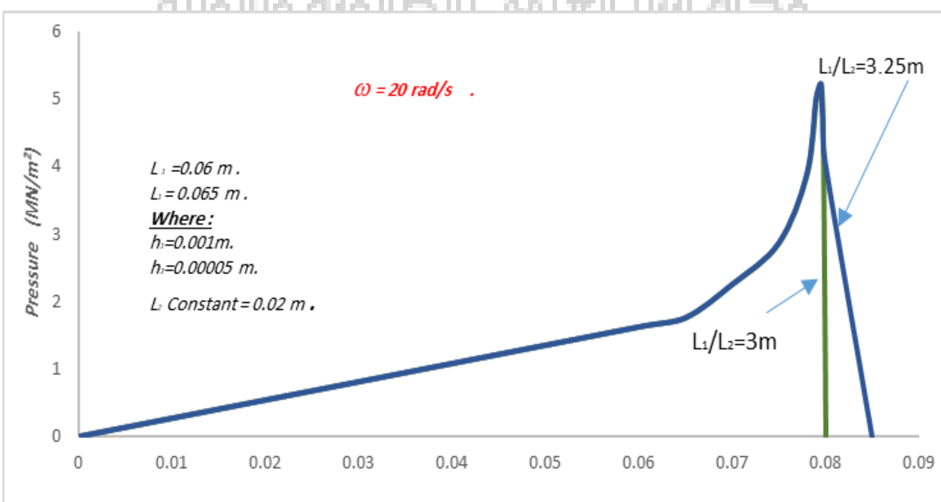

Fig. (5): pressure distribution with multi values.

Fig.(5) shows the pressure distribution with multi values of gap ratios at $\omega = 20$ rad/s and h_1, h_3 is constant.

In Fig(6) the value h_2 , with many values to take the gap ratios h_2/h_3 multiple values are 1.2, 1.4, 2, 6, 8, 10, 16 it could be noted that the maximum pressure value we got to the gap when $h_2/h_3 = 2$, (the critical gap ration) where greater the gap after it, the less pressure.

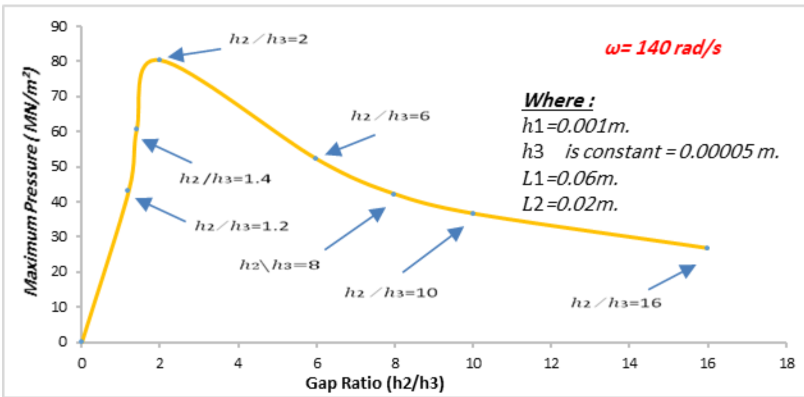

Fig (6) Show the critical gap ratio

Recommendation for further work:

Results indicates that any change in viscosity may results directly in change in hydrodynamic pressure generated, using the set of equations could help using the unit as viscosity measuring device

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